Calc 2 - Test 4

(1) $f(x) = 2x + 3x^{1.7}$

$$F(x) + C = x^2 + \frac{3x^{2.7}}{2.7} + C$$

Check

$F'(x) = 2x + 3x^{1.7}$

(2) $a(t) = 5 + 4t - 2t^2$

$v(0) = 3m/s$

$s(0) = 10m$

$a(t) = v'(t)$

$v(t) = 5t + \frac{4t^2}{2} - \frac{2}{3}t^3 + C$

And $V(0) = 3 = 5(0) + 2(0) - \frac{2}{3}(0) + C$

$\therefore C = 3$

$\therefore v(t) = 5t + 2t^2 - \frac{2}{3}t^3 + 3$

So

$s(t) = \frac{5}{2}t^2 + \frac{2}{3}t^3 - \frac{2}{12}t^4 + 3t + C$

$s(0) = \frac{5}{2}(0) + \frac{2}{3}(0) - \frac{2}{12}(0) + 3(0) + C = 10$

$C = 10$

Calc 2 1

1. $f(x) = 2x + 3x^{1.7}$

$$F(x) + C = x^2 + \frac{3x^{2.7}}{2.7} + C$$

Check

$F'(x) = 2x + 3x^{1.7}$

2. $a(t) = 5 + 4t - 2t^2$

$v(0) = 3\,m/s$

$s(0) = 10\,m$

$a(t) = v'(t)$

$v(t) = 5t + \frac{4}{2}t^2 - \frac{2}{3}t^3 + C$

And $v(0) = 3 = 5(0) + 2(0) - \frac{2}{3}(0) + C$

$\therefore C = 3$

$\therefore v(t) = 5t + 2t^2 - \frac{2}{3}t^3 + 3$

So

$s(t) = \frac{5}{2}t^2 + \frac{2}{3}t^3 - \frac{2}{12}t^4 + 3t + C$

$s(0) = \frac{5}{2}(0) + \frac{2}{3}(0) - \frac{2}{12}(0) + 3(0) + C = 10$

$C = 10$

$$\therefore\ s(t) = -\frac{1}{6}t^4 + \frac{2}{3}t^3 + \frac{5}{2}t^2 + 3t + 10$$

③ a) For right endpoints

$$\int_0^6 f(x)\,dx = \sum_{i=1}^{6} f(x_i)\,\Delta x$$

and since the interval width = 1 (or $\Delta x = 1$)

we get

$$1\,(9.0 + 8.3 + 6.5 + 2.3 - 7.6 - 10.5)$$

$$= \underline{8}$$

b) For left endpoint

$$\sum_{i=1}^{6} f(x_{i-1})\,\Delta x$$

$$= 1\,(9.3 + 9.0 + 8.3 + 6.5 + 2.3 - 7.6)$$

$$= \underline{27.8}$$

c) for midpoints

$$\int_0^6 f(x)\,dx = \sum_{i=1}^{6} f(\bar{x}_i)\,\Delta x$$

$\Delta x = 1$ still

$$\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i)$$

i	1	2	3	4	5	6
$\bar{x}_i$	9.15	8.65	7.4	~~2.65~~ 4.4	-2.65	-9.05

$\therefore$ $1(9.15 + 8.65 + 7.4 + 4.4 - 2.65 - 9.05)$

$= \underline{17.9}$

* Note - Since decreasing function (or so it appears) the right endpoints are too small, left too large, and midpoint more accurate.

c) For midpoints

$$\int_0^6 f(x)\,dx = \sum_{i=1}^{6} f(\bar{x}_i)\,\Delta x$$

$\Delta x = 1$ still

$$\bar{x}_i = \frac{1}{2}\left(x_{i-1} + x_i\right)$$

i	1	2	3	4	5	6
$\bar{x}_i$	9.15	8.65	7.4	4.4	-2.65	-9.05

$$\therefore\ 1\,(9.15 + 8.65 + 7.4 + 4.4 - 2.65 - 9.05)$$

$$= \underline{17.9}$$

* Note - Since decreasing function (or so it appears) the right endpoints are too small, left too large, and midpoint more accurate.

(4.) $\lim_{n \to \infty} \sum_{1}^{n} \frac{e^x}{1 \times x_i} \Delta x$, $[1, 5]$

$$\Rightarrow \int_1^5 \frac{e^x}{1+x}\, dx$$

(5.) a) $\int_{\pi}^{2\pi} \cos\theta \, d\theta$

$$\sin\theta \Big]_{\pi}^{2\pi} = \sin 2\pi - \sin\pi$$

$$= \emptyset$$

b) $\int_0^2 (x^3 - 1)^2 \, dx$

~~$\frac{1}{3}(x^3-1)^3$~~

$$= \int_0^2 x^6 - 2x^3 + 1 \quad dx$$

$$= \frac{1}{7}x^7 - \frac{2}{4}x^4 + x \Big]_0^2$$

$$= \left(\frac{2^7}{7} - \frac{2^4}{2} + 2\right) - 0$$

$$= \frac{128}{7} - \frac{16}{2} + 2 = \boxed{12.28}$$

④ $\lim_{n \to \infty} \sum_{1}^{n} \frac{e^x}{1+x} \Delta x$, $[1, 2]$

$\Rightarrow \int_1^2 \frac{e^x}{1+x} \, dx$

⑤ a) $\int_{\pi}^{2\pi} \cos\theta \, d\theta$

$\sin\theta \Big]_{\pi}^{2\pi} = \sin 2\pi - \sin \pi$

$= 0$

b) $\int_0^2 (v^3 - 1)^2 \, dx$

~~[illegible]~~

$= \int_0^2 x^6 - 2x^3 + 1 \, dx$

$= \frac{1}{7}x^7 - \frac{2}{4}x^4 + x \Big]_0^2$

$= \left(\frac{2^7}{7} - \frac{2^4}{4} + 2 \right) - 0$

$= \frac{128}{7} - \frac{16}{2} + 2 = \boxed{12.28}$

(6) $200 + 50t$ / yr

This is the "rate" of increase, so to get actual increase

$$\int_4^{10} 200 + 50t \; dt$$

$$= 200t + \frac{50}{2}t^2 \Big]_4^{10}$$

$$= (2000 + 2500) - (800 + 400)$$

$$\boxed{= 3300}$$

~~(7) a) $\int x e^{x^2} dx$~~

~~$u = x^2$ $\quad \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$~~

~~$\Rightarrow \int x \, e^u \frac{du}{2x} = \int \frac{e^u}{2} du$~~

~~$= \frac{e^u}{2u} = \frac{1}{2} \frac{1}{x^2} e^{x^2}$~~

~~$= \frac{e^{x^2}}{2x^2}$~~

(7) a) $\int x e^{x^2} dx$

$u = x^2$
$du = 2x\, dx$

$\therefore \int x\, e^u \frac{du}{2x} = \int \frac{e^u}{2} du$

$= \frac{e^u}{2}$

$\Rightarrow \boxed{\frac{e^{x^2}}{2}}$

Test $\frac{d}{dx}\left(\frac{e^{x^2}}{2}\right) = \left(\frac{1}{2} e^{x^2}\right) \cdot 2x$

$= x e^{x^2}$ ✓

(8) b) $\int \cos x \cos(\sin x)\, dx$

$u = \sin x$
$du = \cos x\, dx$

$\Rightarrow \int \cos x \cos u \frac{du}{\cos x} = \int \cos u\, du$

$= \sin u \Rightarrow \boxed{\sin(\sin x)}$

3) $\int x e^{x^2} dx$

$u = x^2$

$du = 2x\, dx$

$\therefore \int x e^{u} \frac{du}{2x} = \int \frac{e^u}{2} du$

$= \frac{e^u}{2}$

$\boxed{\Rightarrow \frac{e^{x^2}}{2}}$

Test $\frac{d}{dx}\left(\frac{e^{x^2}}{2}\right) = \left(\frac{1}{2} e^{x^2}\right) \cdot 2x$

$= x e^{x^2}$ ✓

4) $\int \cos x \cos(\sin x)\, dx$

$u = \sin x$

$du = \cos x\, dx$

$\Rightarrow \int \cos x \cos u \frac{du}{\cos x} = \int \cos u\, du$

$= \sin u \Rightarrow \boxed{\sin(\sin x)}$

(8) a)

$$\int x^4 \ln x \, dx$$

$$= \int f(x) g'(x)\, dx = f(x) g(x) - \int g(x) f'(x)\, dx$$

$f(x) = \ln x \qquad g(x) = \frac{x^5}{5} \qquad g'(x) = x^4$

$$\Rightarrow \int \ln x \, x^4 \, dx = \ln x \frac{x^5}{5} - \int \frac{x^5}{5} \frac{1}{x} \, dx$$

$$= \ln x \frac{x^5}{5} - \int \frac{x^4}{5} \, dx$$

$$= \frac{x^5}{5} \ln x - \frac{x^5}{25}$$

$$\boxed{= \frac{x^5}{5} \left(\ln x - \frac{1}{5} \right)}$$

b) $\int_1^4 \sqrt{t} \ln t \, dt = \int_1^4 dv \, u$

$u = \ln t \qquad dv = \sqrt{t} \qquad v = t^{3/2}$

$$\int_1^4 u \, dv = uv\Big|_1^4 - \int_1^4 v \, du$$

$$= \ln t \sqrt{t}\Big|_1^4 - \int_0^4 t^{3/2} \frac{1}{t} \Rightarrow \int_1^4 t^{1/2}$$

$$= \ln t \sqrt{t}\Big]_1^4 - \frac{2t^{3/2}}{3}\Big]_1^4$$

$=$ ~~tnt~~

(8) a) $\int x^4 \ln x \, dx$

$= \int f(x) g'(x)\, dx = f(x)g(x) - \int g(x) f'(x)\, dx$

$f(x) = \ln x \quad g(x) = \frac{x^5}{5} \quad g'(x) = x^4$

$\Rightarrow \int \ln x \, x^4 dx = \ln x \frac{x^5}{5} - \int \frac{x^5}{5} \frac{1}{x} dx$

$= \ln x \frac{x^5}{5} - \int \frac{x^4}{5} dx$

$= \frac{x^5 \ln x}{5} - \frac{x^5}{25}$

$= \frac{x^5}{5}\left(\ln x - \frac{1}{5}\right)$

b) $\int_1^4 \sqrt{t} \ln t \, dt = \int_1^4 du\, u$

$u = \ln t \quad dv = \sqrt{t} \quad v = t^{3/2}$

$\int_1^4 u\, dv = uv\Big|_1^4 - \int_1^4 v\, du$

$= \ln t \sqrt{t}\Big|_1^4 - \int_1^4 t^{3/2} \frac{1}{t} \Rightarrow \int_1^4 t^{1/2}$

$= \ln t \sqrt{t}\Big]_1^4 - \frac{2t^{3/2}}{3}\Big]_1^4$

$=$

$$\Rightarrow \ln t \sqrt{t}\Big]_1^4 - \frac{2}{3} t^{3/2}\Big]_1^4$$

$$= \left[(\ln 4\sqrt{4}) - (\ln 1 \sqrt{1})\right] - \left[\left(\frac{2}{3} 4^{3/2}\right) - \left(\frac{2}{3} 1^{3/2}\right)\right]$$

$$= (2(1.39) - \emptyset) - \left[\frac{16}{3} - \frac{2}{3}\right]$$

$$= 2.78 - 4.67 = \underline{\underline{-1.894}}$$

(9) a) $\int_0^{\pi/2} \cos^5 x \, dx$

$$= \int \cos^2 x \cos^2 x \cos x \, dx$$

$$= \int (1 - \sin^2 x)^2 \cos x \, dx$$

$u = \sin x$
$du = \cos x \, dx$

$$\int (1 - u^2)^2 \, du = \int 1 - 2u^2 + u^4 \, du$$

$$= u - \frac{2u^3}{3} + \frac{4u^5}{5} \Big]$$

$$= \sin x - \frac{2}{3}\sin^3 x + \frac{4}{5}\sin^5 x \Big]_0^{\pi/2}$$

8

$\Rightarrow \ln t \, dt \Big]_1^4 - \frac{2}{3} t^{3/2} \Big]_1^4$

$= \left[(\ln 4 \cdot 4) - (\ln 1 \cdot 1) \right] - \left[\left(\frac{2}{3} 4^{3/2}\right) - \left(\frac{2}{3} 1^{3/2}\right) \right]$

$= (2(1.38) - 0) - \left[\frac{16}{3} - \frac{2}{3} \right]$

$= 2.76 - 4.67 = -1.894$

(9) a) $\int_0^{\pi/2} \cos^5 x \, dx$

$= \int \cos^2 x \cos^2 x \cos x \, dx$

$= \int (1 - \sin^2 x)^2 \cos x \, dx$

$u = \sin x$
$du = \cos x \, dx$

$\int (1 - u^2)^2 \, du = \int 1 - 2u^2 + u^4 \, du$

$= u - \frac{2u^3}{3} + \frac{u^5}{5} \Big|$

$= \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x \Big]_0^{\pi/2}$

$$= \left(1 - \frac{2}{3} + \frac{4}{5}\right) - 0 = \frac{15}{15} - \frac{10}{15} + \frac{12}{15}$$ 9

$$\boxed{= 17/15 = 1.13}$$

b) $\displaystyle\int_0^1 \frac{x-1}{x^2+3x+2}\,dx$

PFE

$$\frac{x-1}{x^2+3x+2} = \frac{A}{x+2} + \frac{B}{x+1}$$

$$x-1 = A(x+1) + B(x+2)$$
$$x-1 = Ax + A + Bx + 2B$$
$$x-1 = x(A+B) + (A+2B)$$

$$A + 2B = -1 \checkmark$$
$$A + B = 1 \checkmark$$
$$B = -2$$

$$\therefore A = 3$$

$$\int_0^1 \frac{3}{x+2} + \frac{-2}{x+1}\,dx$$

$$= 3\ln|x+2| - 2\ln|x+1| \Big]_0^1$$
$$(9.89 - 1.38) - 2.08 = \boxed{6.42}$$

(10)

a) $\int e^{2\theta} \sin(3\theta)\, d\theta$

from # 98 on table

$$= \frac{e^{2\theta}}{2^2+3^2} (2 \sin 3\theta - 3 \cos 3\theta) + C$$

$$= \frac{e^{2\theta}}{13} (2 \sin 3\theta - 3 \cos 3\theta) + C$$

b) $\int x^2 \tan^{-1}(x)\, dx$ # 95 on table

$$= \frac{1}{2+1} \left[x^{2+1} \tan^{-1} x - \int \frac{x^{2+1}}{1+x^2} dx \right]$$

$$= \frac{1}{3} \left[x^3 \tan^{-1} x - \int \frac{x^3}{1+x^2} dx \right]$$

(10) a) $\int e^{2\theta} \sin(3\theta)\, d\theta$

from # 98 on table

$= \frac{e^{2\theta}}{2^2+3^2} \left(2\sin 3\theta - 3\cos 3\theta \right) + C$

$= \frac{e^{2\theta}}{13} \left(2\sin 3\theta - 3\cos 3\theta \right) + C$

b) $\int x^2 \tan^{-1}(x)\, dx$ # 92 on table

$= \frac{1}{2+1} \left[x^{2+1} \tan^{-1} x - \int \frac{x^{2+1}}{1+x^2}\, dx \right].$

$= \frac{1}{3} \left[x^3 \tan^{-1} x - \int \frac{x^3}{1+x^2}\, dx \right]$

Calculus Test #5

1) Convergent or Divergent — 1/7/11

$$\int_{-\infty}^{0} \frac{1}{2x-5}\,dx = \lim_{t\to-\infty}\int_{t}^{0} \frac{1}{2x-5}\,dx$$

Let $u = 2x-5$ $\quad du = 2dx$

$$\int \frac{1}{u}\frac{du}{2} = \frac{\ln|u|}{2}$$

$$\Rightarrow \lim_{t\to-\infty} \left.\frac{\ln|2x-5|}{2}\right]_{t}^{0}$$

$$= \lim_{t\to-\infty}\left(\frac{\ln|-5|}{2} - \frac{\ln|2t-5|}{2}\right)$$

$$= \lim_{t\to-\infty}\left(\frac{\ln 5}{2} - \frac{\ln|2t-5|}{2}\right)$$

~ 0.8 $\quad \to \infty$

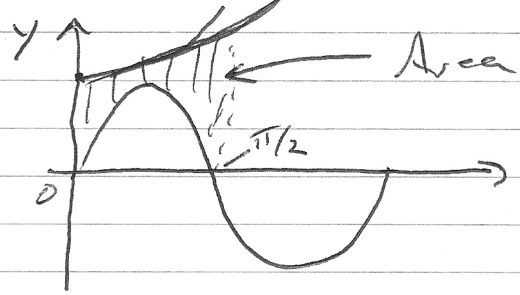
Divergent

~~6.1#6~~

2) Sketch the region enclosed ..... Find Area

(6.1#6)

$y = \sin x$, $y = e^x$, $x = 0$ $x = \pi/2$



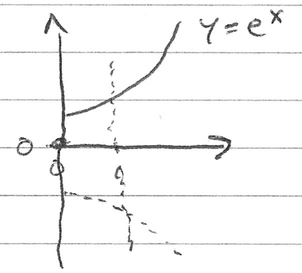
∴ To get area ~~integrate~~ integrate wrt x

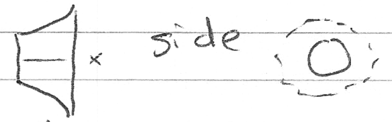
$$\int_0^{\pi/2} (e^x - \sin x)\,dx = \int e^x\,dx - \int \sin x\,dx$$

$$= e^x - (-\cos x)\Big]_0^{\pi/2} = (e^{\pi/2} + 0) - (1 + 1)$$

$$= e^{\pi/2} - 2$$

3)





$A = \pi r^2 \quad r = e^x$

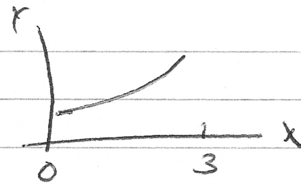
$$V = \int_0^1 \pi (e^x)^2\,dx \quad \sout{= \pi \int_0^1 e^{2x}\,dx}$$

$$= \sout{\pi} \left(\frac{\pi e^{2x}}{2}\right)\Big|_0^1 = \sout{\pi\left(e^{2x}\right)}$$

$$= \pi\left(\frac{e^2}{2} - \frac{1}{2}\right) = \frac{\pi}{2}\left(e^2 - 1\right)$$

4)

$y = 2^x \qquad 0 \le x \le 3$



$$L = \int_0^3 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = 2^x = 2x \ln 2$$

~~$L = \int_0^3 \sqrt{1 + \frac{dy}{}}$~~

$$L = \int_0^3 \sqrt{1 + (2x\ln 2)^2}\, dx$$

5) $f(x) = \ln x \quad [1, 3]$

a) $f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2}\int_1^3 \ln x\, dx$

$= \frac{1}{2}\Big[x\ln x - x\Big]_1^3 = \frac{1}{2}\Big[(3\ln 3 - 3)$

$- (\ln 1 - 1)\Big]$

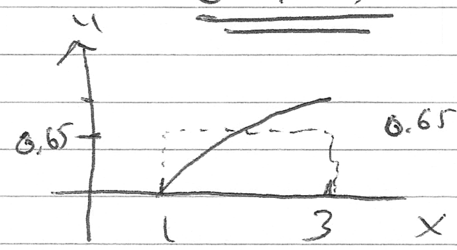
$= \frac{1}{2}\big(0.3 - (-1)\big) = 0.65$

b) find c so $f_{AVG} = f(c)$

$f(c) = \ln c = 0.65 =$

$\underline{\underline{c = 1.91}}$

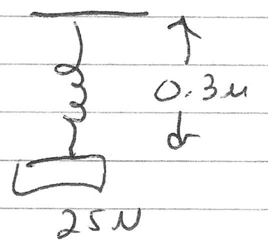
c)



~~Total Area = 2 x 0.65~~

=

6) Spring Natural Length = 20 cm = 0.2 m



$f = Kx$

$25N = k(0.1m)$

$\underline{K = 250}$

$$W = \int_{0.2}^{0.25} f(x)\,dx \int_{0.2}^{0.25} 250x\,dx = \left[\frac{250}{2}x^2\right]_{0.2}^{0.25}$$

$$= 125\ (0.0225) = 2.81 \text{ Joules}$$

7) $\frac{P}{P_0} = \left(\frac{R_0}{R}\right)^4$ $\qquad F = \frac{\pi P R^4}{8\eta l}$

$F_0 = \frac{\pi P_0 R_0^4}{8\eta l}$ $\qquad F = \frac{\pi P R^4}{8\eta l}$

If $F_0 = F$

$\frac{\pi P_0 R_0^4}{8\eta l} = \frac{\pi P R^4}{8\eta l}$

$\Rightarrow \frac{P}{P_0} = \left(\frac{R_0}{R}\right)^4$ ✓ Part a

Now if $R = \frac{3}{4}R_0$ then

$\frac{P}{P_0} = \left(\frac{R_0}{\frac{3}{4}R_0}\right)^4$ $\qquad \Rightarrow P = P_0\left(\frac{4}{3}\right)^4$

$\boxed{P = P_0 \cdot 3.16}$

Part b

8) Verify

$y = \sin x \cos x - \cos x$ is a solution to

$y' + (\tan x)y = \cos^2 x \quad y(0) = -1$ on $-\pi/2 < x < \pi/2$

$\frac{dy}{dx} = y' =$ ~~cos x cos x + sin x sin x − sin x~~

$= (\cos x \cos x + \sin x(-\sin x)) - (-\sin x)$

$= \cos^2 x - \sin^2 x + \sin x$

$=$

So $\cos^2 x - \sin^2 x + \sin x + (\tan x)(\sin x \cos x - \cos x)$

$= \cos^2 x$

~~y(0) = ? 1 − 0 + 0 + (0)(...) y(0) = 1~~

So $y' = \cos^2 x - \tan x(\sin x \cos x - \cos x) =$ ~~cos² x~~

$y(0) = -1 \stackrel{?}{=} \sin(0)\cos(0) - \cos(0) = -1$ ✓

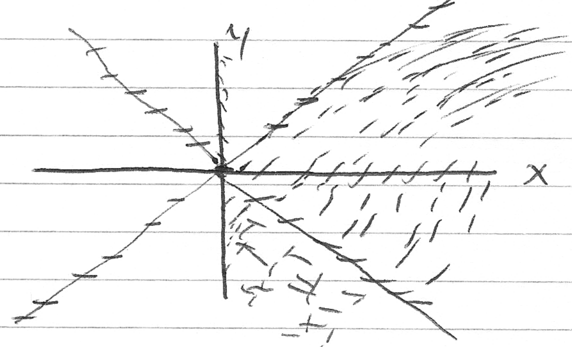
So $y' = \cos^2 x - \frac{\sin x}{\cos x}(\sin x \cos x - \cos x)$ ~~= cos x~~

$= \cos^2 x - \sin^2 x + \sin x$ ✓

SAME

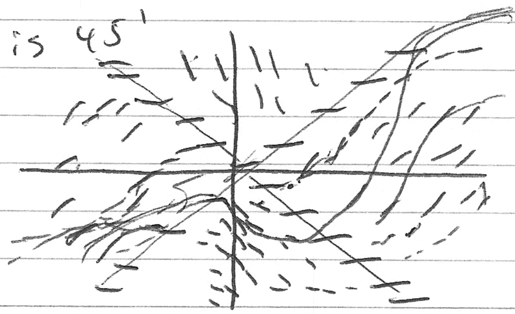
9) $y' = x^2 - y^2$

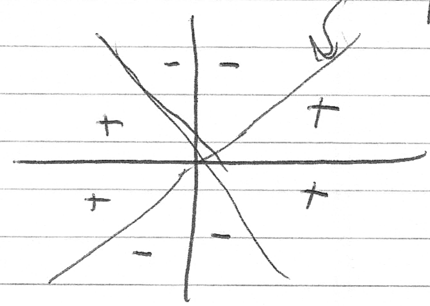
Sketch direction of diff Equation



y'	x	y
0	0	0

Key is 45'





10) $\frac{dy}{dt} = te^{y} \quad y(1) = \emptyset$

$\int dy = \int te^{y}\, dt \quad y(1) = 0$

$y + C =$

$\frac{1}{e^{y}}\, dy = t\, dt$

$\int \frac{1}{e^{y}}\, dy = \int t\, dt$

$-e^{-y} + C_1 = \frac{1}{2}t^2 + C_2 \qquad y(1) = \emptyset$

$t = 1 \Rightarrow y = \emptyset$

$-e^{-y} + C_1 = \frac{1}{2}(1)^2 + C_2$

$-1 + C_1 = \frac{1}{2} + C_2$

$C_2 = -1\frac{1}{2} = -1.5 = -3/2$

$\therefore -e^{-y} = \frac{1}{2}t^2 - 3/2$