

# Calc 2 - Test 4

①  $f(x) = 2x + 3x^{1.7}$

$$F(x) + C = x^2 + \frac{3x^{2.7}}{2.7} + C$$

Check

$$F'(x) = 2x + 3x^{1.7}$$

②  $a(t) = 5 + 4t - 2t^2$

$$v(0) = 3 \text{ m/s}$$

$$s(0) = 10 \text{ m}$$

$$a(t) = v'(t)$$

$$v(t) = 5t + \frac{4t^2}{2} - \frac{2}{3}t^3 + C$$

$$\text{And } v(0) = 3 = 5(0) + 2(0) - \frac{2}{3}(0) + C$$

$$\therefore C = 3$$

$$\therefore v(t) = 5t + 2t^2 - \frac{2}{3}t^3 + 3$$

So

$$s(t) = \frac{5}{2}t^2 + \frac{2}{3}t^3 - \frac{2}{12}t^4 + 3t + C$$

$$s(0) = \frac{5}{2}(0) + \frac{2}{3}(0) - \frac{2}{12}(0) + 3(0) + C = 10$$

$$C = 10$$

Calc 2 - Test 1

①  $z(x) = 2x + 3x^{1/2}$

$$F(x) + C = x^2 + 3x^{3/2} + C$$

Check

$$F'(x) = 2x + 3x^{1/2}$$

②  $z(t) = 2t + 4t - 2t^2$

$$v(0) = 3 \text{ m/s}$$

$$z(0) = 10 \text{ m}$$

$$a(t) = v'(t)$$

$$v(t) = 2t + 4t - \frac{2}{3}t^3 + C$$

$$\text{And } v(0) = 3 = z(0) + 3(0) - \frac{2}{3}(0) + C$$

$$\therefore C = 3$$

$$\therefore v(t) = 2t + 4t - \frac{2}{3}t^3 + 3$$

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$$z(t) = \frac{2}{3}t^3 + \frac{4}{2}t^2 + 3t + C$$

$$z(0) = \frac{2}{3}(0) + \frac{4}{2}(0) + 3(0) + C = 10$$

$$C = 10$$

$$S(t) = -\frac{1}{6}t^4 + \frac{2}{3}t^3 + \frac{5}{2}t^2 + 3t + 10$$

③ a) For right endpoints

$$\int_0^6 f(x) dx = \sum_{i=1}^6 f(x_i) \Delta x$$

and since the interval width = 1 (or  $\Delta x = 1$ )

We get

$$\begin{aligned} & 1 (9.0 + 8.3 + 6.5 + 2.3 - 7.6 - 10.5) \\ & = \underline{\underline{8}} \end{aligned}$$

b) For left endpoint

$$\sum_{i=1}^6 f(x_{i-1}) \Delta x$$

$$\begin{aligned} & = 1 (9.3 + 9.0 + 8.3 + 6.5 + 2.3 - 7.6) \\ & = \underline{\underline{27.8}} \end{aligned}$$

$$\int_0^1 (2x^2) dx = \frac{2}{3}x^3 + \frac{1}{2}x^2 - \frac{1}{3}x + 10$$

③ For left endpoint

$$\sum_{i=1}^n f(x_{i-1}) \Delta x = \int_0^1 f(x) dx$$

and since the interval width = 1 (or  $\Delta x = 1$ )

we get

$$1 (0.0 - 0.3 + 0.2 + 0.7 - 1.0)$$

$$= 0$$

④ For left endpoint

$$\sum_{i=1}^n f(x_{i-1}) \Delta x$$

$$1 (0.7 + 0.0 + 0.3 + 0.2 + 0.7 - 1.0)$$

$$= 0.5$$

c) for midpoints

$$\int_0^6 f(x) dx = \sum_{i=1}^6 f(\bar{x}_i) \Delta x$$

$$\Delta x = 1 \text{ still}$$

$$\bar{x}_i = \frac{1}{2} (x_{i-1} + x_i)$$

$i$	1	2	3	4	5	6
$\bar{x}_i$	9.15	8.65	7.4	<del>3.65</del> 4.4	-2.65	-9.05

$$\approx 1 (9.15 + 8.65 + 7.4 + 4.4 - 2.65 - 9.05)$$

$$= \underline{\underline{17.9}}$$

\* Note - Since decreasing function (or so it appears) the right endpoints are too small, left too large, and midpoint more accurate.

c) for midpoint

$$\Delta x \left( \bar{x}_i \right) \approx \sum_{i=1}^n f(x_i) \Delta x$$

$$\Delta x = 1 - x_{i-1}$$

$$\bar{x}_i = \frac{1}{2} (x_{i-1} + x_i)$$

0	2	4	6	8	10	12
20.9	22.5	<del>24.1</del>	25	26.8	28.2	29

2.4

$$(20.9 - 22.5 - 24.1 + 25 + 26.8 + 28.2 + 29) \Delta x$$

$$17.1 =$$

\* Note - Since decreasing function (or so it appears) the right endpoints are too small, left endpoints are too large, and midpoint more accurate.

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$$(4) \quad \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{e^x}{1+x_i} \Delta x, \quad [1, 5]$$

$$\Rightarrow \int_1^5 \frac{e^x}{1+x} dx$$

$$(5) \quad a) \int_{\pi}^{2\pi} \cos \theta d\theta$$

$$\sin \theta \Big|_{\pi}^{2\pi} = \sin 2\pi - \sin \pi$$

$$= 0$$

$$b) \int_0^2 (\sqrt{3}-1)^2 dx$$

$$= \int_0^2 \frac{1}{3} (\sqrt{3}-1)^3 dx$$

$$= \int_0^2 x^6 - 2x^3 + 1 dx$$

$$= \left[ \frac{1}{7} x^7 - \frac{2}{4} x^4 + x \right]_0^2$$

$$= \left( \frac{2^7}{7} - \frac{2^4}{2} + 2 \right) - 0$$

$$= \frac{128}{7} - \frac{16}{2} + 2 = \boxed{12.28}$$

$$\textcircled{1} \quad \lim_{N \rightarrow \infty} \sum_{n=0}^N \frac{e^{j\omega n}}{1 - e^{j\omega n}} \Delta x \quad [1, 2]$$

$$\Rightarrow \left. \sum_{n=0}^{\infty} \frac{e^{j\omega n}}{1 - e^{j\omega n}} \right\}$$

$$\textcircled{2} \quad \left. \cos \theta \right\}$$

$$= \left[ \frac{e^{j\omega n}}{1 - e^{j\omega n}} \right]_{n=0}^{\infty} = \frac{e^{j\omega n}}{1 - e^{j\omega n}} - \frac{e^{j\omega(n-1)}}{1 - e^{j\omega(n-1)}} = \frac{e^{j\omega n}}{1 - e^{j\omega n}} - \frac{e^{j\omega n}}{1 - e^{j\omega n}}$$

$$\textcircled{3} \quad \left. \frac{e^{j\omega n}}{1 - e^{j\omega n}} \right\}$$

~~$$\frac{1}{1 - e^{j\omega n}}$$~~

$$= \left. \frac{e^{j\omega n}}{1 - e^{j\omega n}} \right\}$$

$$= \left[ \frac{1}{1 - e^{j\omega n}} \right]_{n=0}^{\infty} = \frac{1}{1 - e^{j\omega n}} - \frac{1}{1 - e^{j\omega(n-1)}} = \frac{1}{1 - e^{j\omega n}} - \frac{1}{1 - e^{j\omega n}}$$

$$= \left( \frac{1}{1 - e^{j\omega n}} - \frac{1}{1 - e^{j\omega(n-1)}} \right) = 0$$

$$= \frac{1}{1 - e^{j\omega n}} - \frac{1}{1 - e^{j\omega(n-1)}} = \frac{1}{1 - e^{j\omega n}} - \frac{1}{1 - e^{j\omega n}} = 0$$

$$\textcircled{6} \quad 200 + 50t \quad / \text{yr}$$

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This is the "rate" of increase, so  
to get actual increase

$$\begin{aligned} & \int_4^{10} (200 + 50t) dt \\ &= \left[ 200t + \frac{50}{2}t^2 \right]_4^{10} \\ &= (2000 + 2500) - (800 + 400) \\ &= 3300 \end{aligned}$$

$$\textcircled{7} \quad \text{a) } \int x e^{x^2} dx$$

$$u = x^2 \quad \frac{du}{dx} = 2x \quad \Rightarrow dx = \frac{du}{2x}$$

$$\Rightarrow \int x e^u \frac{du}{2x} = \int \frac{e^u}{2} du$$

$$= \frac{1}{2} e^u = \frac{1}{2} \frac{1}{x^2} e^{x^2}$$

$$= \frac{e^{x^2}}{2x^2}$$

$$\textcircled{2} \quad 300 + 20 = 320 \quad \text{yr}$$

This is the "rate" of increase, 20  
to get actual increase

$$= \left[ 300 + \frac{20}{8} \right]_{\text{yr}} - \left[ 300 + 20 \right]_{\text{yr}}$$

$$= (300 + 20) - (300 + 40)$$

$$= 300$$

~~$$\textcircled{1} \quad a) \quad \left. \begin{aligned} &v = x^2 \cdot q^x \\ &r = x \end{aligned} \right\} \Rightarrow \frac{dv}{dx} = 2x \cdot q^x + x^2 \cdot q^x \ln q$$

$$= \frac{2x \cdot q^x + x^2 \cdot q^x \ln q}{x^2} = \frac{2 + x \ln q}{x}$$~~

$$\textcircled{7} a) \int x e^{x^2} dx$$

6

$$u = x^2$$

$$du = 2x dx$$

$$\therefore \int x e^u \frac{du}{2x} = \int \frac{e^u}{2} du$$

$$= \frac{e^u}{2}$$

$$\Rightarrow \frac{e^{x^2}}{2}$$

$$\text{Test } \frac{d}{dx} \left( \frac{e^{x^2}}{2} \right) = \left( \frac{1}{2} e^{x^2} \right) \cdot 2x$$

$$= x e^{x^2} \checkmark$$

$$\textcircled{8} b) \int \cos x \cos(\sin x) dx$$

$$u = \sin x$$

$$du = \cos x dx$$

$$\Rightarrow \int \cos x \cos u \frac{du}{\cos x} = \int \cos u du$$

$$= \sin u$$

$$\Rightarrow \sin(\sin x)$$

$$\textcircled{7} \quad \left. \begin{aligned} & x b^x e^x \\ & a x \end{aligned} \right\}$$

$$u = x^2$$

$$du = 2x dx$$

$$\int \frac{u^2}{2} = \frac{u^3}{6} + C = \frac{x^6}{6} + C$$

$$\frac{u^3}{6} =$$

$$\int \frac{u^2}{2} = \frac{u^3}{6} + C$$

$$\int \frac{1}{2} \cdot \left( \frac{1}{2} e^x \right) \cdot 2x = \frac{1}{2} \left( \frac{1}{2} e^x \right) = \frac{1}{4} e^x$$

$$= x e^x + C$$

$$\textcircled{8} \quad \left. \begin{aligned} & \cos x \cos(2x) \\ & a x \end{aligned} \right\}$$

$$u = 2x$$

$$du = 2 dx$$

$$\int \cos x \cos(2x) = \frac{1}{2} \int \cos \frac{u}{2} \cos u = \frac{1}{2} \cos u \sin u + C = \frac{1}{2} \cos 2x \sin 2x + C$$

$$\boxed{= \frac{1}{2} \sin(2x) \cos(2x)}$$

8) a)

7

$$\int x^4 \ln x \, dx$$

$$= \int f(x) g'(x) \, dx = f(x) g(x) - \int g(x) f'(x) \, dx$$

$$f(x) = \ln x \quad g(x) = \frac{x^5}{5} \quad g'(x) = x^4$$

$$\Rightarrow \int \ln x x^4 \, dx = \ln x \frac{x^5}{5} - \int \frac{x^5}{5} \frac{1}{x} \, dx$$

$$= \ln x \frac{x^5}{5} - \int \frac{x^4}{5} \, dx$$

$$= \frac{x^5}{5} \ln x - \frac{x^5}{25}$$

$$= \frac{x^5}{5} \left( \ln x - \frac{1}{5} \right)$$

$$b) \int_1^4 \sqrt{t} \ln t \, dt = \int_1^4 du \, u$$

$$\begin{aligned} \int_1^4 \sqrt{t} \ln t \, dt &= \int_1^4 u \, du \\ u &= \ln t \quad dv = \sqrt{t} \quad v = t^{3/2} \end{aligned}$$

$$\int_1^4 u \, dv = uv \Big|_1^4 - \int_1^4 v \, du$$

$$= \ln t \sqrt{t} \Big|_1^4 - \int_1^4 t^{3/2} \frac{1}{t} \, dt \Rightarrow \int_1^4 t^{1/2}$$

$$= \ln t \sqrt{t} \Big|_1^4 - \frac{2t^{3/2}}{3} \Big|_1^4$$

$$= \ln 4 \cdot 2 - \frac{2 \cdot 8}{3} + \frac{2}{3}$$

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3) a)

$$\left\{ \begin{matrix} x^2 + 1 \\ x^2 + 1 \end{matrix} \right\}$$

$$= \left( \begin{matrix} 2x \\ 2x \end{matrix} \right) \cdot \left( \begin{matrix} x \\ x \end{matrix} \right) = \left( \begin{matrix} 2x^2 \\ 2x^2 \end{matrix} \right)$$

$$\frac{2x^2}{2} = x^2 \quad \frac{2x^2}{2} = x^2$$

$$\Rightarrow \left( \begin{matrix} 1 \\ 1 \end{matrix} \right) \cdot \left( \begin{matrix} x \\ x \end{matrix} \right) = \left( \begin{matrix} x \\ x \end{matrix} \right)$$

$$= \frac{1}{2} \cdot \left( \begin{matrix} x \\ x \end{matrix} \right) = \left( \begin{matrix} \frac{x}{2} \\ \frac{x}{2} \end{matrix} \right)$$

$$= \frac{x}{2} - \frac{x}{2} = 0$$

$$\boxed{= \frac{x}{2} (1 - 1)}$$

$$b) \left\{ \begin{matrix} x^2 + 1 \\ x^2 + 1 \end{matrix} \right\}$$

$$m = 1 \quad n = 1 \quad p = 1 \quad q = 1 \quad r = 1$$

$$\left\{ \begin{matrix} m^2 + n^2 \\ m^2 + n^2 \end{matrix} \right\} = \left\{ \begin{matrix} 2 \\ 2 \end{matrix} \right\}$$

$$\Rightarrow \frac{1}{2} \cdot \left( \begin{matrix} 2 \\ 2 \end{matrix} \right) = \left( \begin{matrix} 1 \\ 1 \end{matrix} \right)$$

$$= \frac{1}{2} \cdot \left( \begin{matrix} 2 \\ 2 \end{matrix} \right) = \left( \begin{matrix} 1 \\ 1 \end{matrix} \right)$$

~~1~~

$$\Rightarrow \left[ \ln t \sqrt{t} \right]_1^4 - \left[ \frac{2}{3} t^{3/2} \right]_1^4 \quad \text{②}$$

$$= \left[ (\ln 4 \sqrt{4}) - (\ln 1 \sqrt{1}) \right] - \left[ \left( \frac{2}{3} 4^{3/2} \right) - \left( \frac{2}{3} 1^{3/2} \right) \right]$$

$$= (2(1.39) - 0) - \left[ \frac{16}{3} - \frac{2}{3} \right]$$

$$= 2.78 - 4.67 = \underline{\underline{-1.894}}$$

$$\textcircled{9} \text{ a) } \int_0^{\pi/2} \cos^5 x \, dx$$

$$= \int \cos^2 x \cos^2 x \cos x \, dx$$

$$= \int (1 - \sin^2 x)^2 \cos x \, dx$$

$$u = \sin x$$

$$du = \cos x \, dx$$

$$\int (1 - u^2)^2 \, du = \int 1 - 2u^2 + u^4 \, du$$

$$= u - \frac{2u^3}{3} + \frac{u^5}{5} \quad \updownarrow$$

$$= \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x \Big|_0^{\pi/2}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \cdot 1 \cdot 1) - (1 \cdot 1 \cdot 1) \\ (1 \cdot 1 \cdot 1) - (1 \cdot 1 \cdot 1) \\ (1 \cdot 1 \cdot 1) - (1 \cdot 1 \cdot 1) \end{bmatrix} =$$

$$= \begin{bmatrix} 1 - 1 \\ 1 - 1 \\ 1 - 1 \end{bmatrix} =$$

$$\underline{\underline{0 \cdot 0 \cdot 0 = 0 - 0 - 0 = 0}}$$

$$\textcircled{P} \int_0^{\pi/2} \cos^2 x \, dx$$

$$= \int_0^{\pi/2} \cos^2 x \, dx$$

$$= \int_0^{\pi/2} (1 - 2\sin^2 x) \, dx$$

$$\int_0^{\pi/2} 1 - 2\sin^2 x \, dx$$

$$\left( x - \frac{2}{3} \sin^3 x \right) \Big|_0^{\pi/2} = \left( \frac{\pi}{2} - \frac{2}{3} \sin^3 \frac{\pi}{2} \right) - \left( 0 - \frac{2}{3} \sin^3 0 \right)$$

$$= \frac{\pi}{2} - \frac{2}{3} \sin^3 \frac{\pi}{2} + \frac{2}{3} \sin^3 0$$

$$= \frac{\pi}{2} - \frac{2}{3} \cdot 1 + \frac{2}{3} \cdot 0 = \frac{\pi}{2} - \frac{2}{3}$$

$$= \left(1 - \frac{2}{3} + \frac{4}{5}\right) - 0 = \frac{15}{15} - \frac{10}{15} + \frac{12}{15} \quad 9$$

$$= \frac{17}{15} = 1.13$$

$$b) \int_0^1 \frac{x-1}{x^2+3x+2} dx$$

PFE

$$\frac{x-1}{x^2+3x+2} = \frac{A}{x+2} + \frac{B}{x+1}$$

$$x-1 = A(x+1) + B(x+2)$$

$$x-1 = Ax + A + Bx + 2B$$

$$x-1 = x(A+B) + (A+2B)$$

$$A+2B = -1 \quad \checkmark$$

$$A+B = 1 \quad \checkmark$$

$$B = -2$$

$$\therefore A = 3$$

$$\int_0^1 \frac{3}{x+2} + \frac{-2}{x+1} dx$$

$$= 3 \ln|x+2| - 2 \ln|x+1| \Big|_0^1 = \boxed{6.42}$$

~~8.8~~

$$I \quad \frac{4}{12} + \frac{10}{12} - \frac{12}{12} = 0 \quad \left( 1 - \frac{4}{12} + \frac{4}{12} \right) =$$

$$\boxed{\frac{1}{12} = 1 \cdot 12}$$

$$P) \quad \frac{x-1}{x^2+3x+2} \quad \text{or} \quad \frac{x-1}{(x+1)(x+2)}$$

PDF

$$\frac{A}{x+1} + \frac{B}{x+2} = \frac{x-1}{(x+1)(x+2)}$$

$$x-1 = A(x+1) + B(x+2)$$

$$x-1 = Ax + A + Bx + 2B$$

$$x-1 = x(A+B) + (A+2B)$$

$$1 + 2B = -1$$

$$1 = B + A$$

$$B = -2$$

$$A = 3$$

$$\frac{3}{x+1} + \frac{-2}{x+2}$$

$$\int \left( \frac{3}{x+1} - \frac{2}{x+2} \right) dx = 3 \ln|x+1| - 2 \ln|x+2| + C$$

$$(10) \quad a) \int e^{2\theta} \sin(3\theta) d\theta$$

from #19 on table

$$= \frac{e^{2\theta}}{2^2 + 3^2} (2 \sin 3\theta - 3 \cos 3\theta) + C$$

$$= \frac{e^{2\theta}}{13} (2 \sin 3\theta - 3 \cos 3\theta) + C$$

$$b) \int x^2 \tan^{-1}(x) dx \quad \# 25 \text{ on table}$$

$$= \frac{1}{2+1} \left[ x^{2+1} \tan^{-1} x - \int \frac{x^{2+1} dx}{1+x^2} \right]$$

$$= \frac{1}{3} \left[ x^3 \tan^{-1} x - \int \frac{x^3 dx}{1+x^2} \right]$$

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$$\left. \begin{aligned} &e^{3s} (2s^2 - 3\cos 3\theta) + C \\ &e^{3s} (2s^2 - 3\cos 3\theta) + C \end{aligned} \right\} \text{part 1 \& part 2}$$

$$= \frac{e^{3s}}{s^2 + 3s} (2s^2 - 3\cos 3\theta) + C$$

$$= \frac{e^{3s}}{s} (2s - 3\cos 3\theta) + C$$

$$\left. \begin{aligned} &x^2 \tan^{-1}(x) + C \\ &x^2 \tan^{-1}(x) + C \end{aligned} \right\} \text{part 1 \& part 2}$$

$$= \frac{1}{s+1} [x^2 \tan^{-1}(x) - \frac{x^2}{1+x^2}] + C$$

$$= \frac{1}{3} [x^2 \tan^{-1}(x) - \frac{x^2}{1+x^2}] + C$$

# Calculus Test #5

1) Convergent or Divergent

1/7/11

$$\int_{-\infty}^0 \frac{1}{2x-5} dx = \lim_{t \rightarrow -\infty} \int_t^0 \frac{1}{2x-5} dx$$

$$\text{Let } u = 2x-5 \quad du = 2dx$$

$$\int \frac{1}{u} \frac{du}{2} = \frac{\ln|u|}{2}$$

$$\Rightarrow \lim_{t \rightarrow -\infty} \left. \frac{\ln|2x-5|}{2} \right|_t^0$$

$$= \lim_{t \rightarrow -\infty} \left( \frac{\ln|5|}{2} - \frac{\ln|2t-5|}{2} \right)$$

$$= \lim_{t \rightarrow -\infty} \left( \frac{\ln 5}{2} - \frac{\ln|2t-5|}{2} \right)$$

$\swarrow$   
 $\approx 0.8$

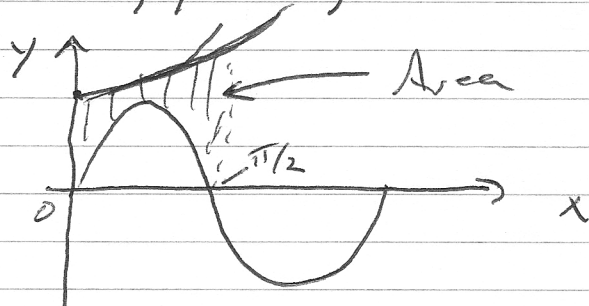
$\searrow$   
 $\rightarrow \infty$

Divergent

~~6.1#6~~

2) Sketch the region enclosed ..... Find Area (6.1#6)

$$y = \sin x, y = e^x, x=0, x = \pi/2$$



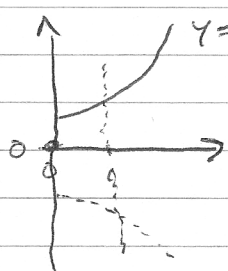
To get area ~~integrate~~ integrate wrt x

$$\int_0^{\pi/2} (e^x - \sin x) dx = \int e^x dx - \int \sin x dx$$

$$= e^x - (-\cos x) \Big|_0^{\pi/2} = (e^{\pi/2} + 0) - (1 + 0)$$

$$= e^{\pi/2} - 2$$

3)



side  $\odot$

$$A = \pi r^2 \quad r = e^x$$

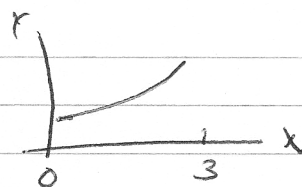
$$V = \int_0^1 \pi (e^x)^2 dx = \pi \int_0^1 e^{2x} dx$$

$$= \pi \left( \frac{e^{2x}}{2} \right)_0^1 = \pi \frac{e^2 - 1}{2}$$

$$= \pi \left( e^{2/2} - \frac{1}{2} \right) = \frac{\pi}{2} (e^2 - 1)$$

4)

$$y = 2^x \quad 0 \leq x \leq 3$$



$$L = \int_0^3 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$\frac{dy}{dx} = 2^x = 2x \ln 2$$

~~$$L = \int_0^3 \sqrt{1 + \frac{dy}{dx}} dx$$~~

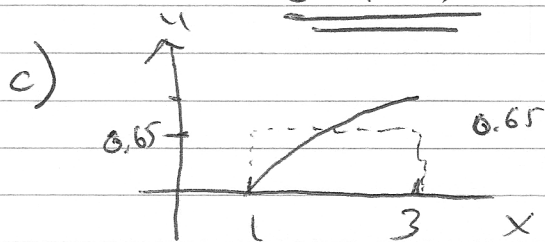
$$L = \int_0^3 \sqrt{1 + (2x \ln 2)^2} dx$$

5)  $f(x) = \ln x \quad [1, 3]$

$$\begin{aligned} c) f_{\text{avg}} &= \frac{1}{b-a} \int_a^b f(x) dx = \frac{1}{2} \int_1^3 \ln x dx \\ &= \frac{1}{2} \left[ x \ln x - x \right]_1^3 = \frac{1}{2} \left[ (3 \ln 3 - 3) \right. \\ &\quad \left. - (\ln 1 - 1) \right] \\ &= \frac{1}{2} (0.3 - (-1)) = 0.65 \end{aligned}$$

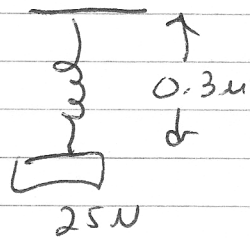
$$D) \text{ find } c \text{ so } f_{\text{avg}} = f(c)$$

$$\begin{aligned} f(c) &= \ln c = 0.65 = \\ &\quad \underline{\underline{c = 1.91}} \end{aligned}$$



~~$$\text{Total Area} = 2 \times 0.65 =$$~~

b) Spring Natural Length = 20 cm = 0.2 m



$$f = kx$$

$$25N = k(0.1m)$$

$$\underline{k = 250}$$

$$W = \int_{0.2}^{0.25} f(x) dx = \int_{0.2}^{0.25} 250x dx = \left[ \frac{250}{2} x^2 \right]_{0.2}^{0.25}$$

$$= 125 (0.0225) = 2.81 \text{ Joules} \checkmark$$

$$\Rightarrow \frac{P}{P_0} = \left( \frac{R_0}{R} \right)^4 \quad F = \frac{\pi P R^4}{8nl}$$

$$F_0 = \frac{\pi P_0 R_0^4}{8nl} \quad F = \frac{\pi P R^4}{8nl}$$

$$\text{If } F_0 = F$$

$$\frac{\pi P_0 R_0^4}{8nl} = \frac{\pi P R^4}{8nl}$$

$$\Rightarrow \frac{P_0}{P} = \left( \frac{R_0}{R} \right)^4 \quad \checkmark \quad \underline{\text{Part a}}$$

Now if  $R = \frac{3}{4} R_0$  then

$$\frac{P}{P_0} = \left( \frac{R_0}{\frac{3}{4} R_0} \right)^4 \quad \Rightarrow P = P_0 \left( \frac{4}{3} \right)^4$$

$$\boxed{P = P_0 \cdot 3.16}$$

Part b

8) Verify

$y = \sin x \cos x - \cos x$  is a solution to

$$y' + (\tan x)y = \cos^2 x \quad y(0) = -1 \quad \text{on } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$\begin{aligned} \frac{dy}{dx} = y' &= \overline{-\cos x \cos x + \sin x \sin x - \sin x} \\ &= (\cos x \cos x + \sin x (-\sin x)) - (-\sin x) \\ &= \cos^2 x - \sin^2 x + \sin x \\ &= \end{aligned}$$

$$\begin{aligned} \circ \circ \quad \cos^2 x - \sin^2 x + \sin x + (\tan x) (\sin x \cos x - \cos x) \\ = \cos^2 x \end{aligned}$$

~~$y(0) = -1 = 1 - 0 + 0 + (0)(1 - 1) = 1$~~

$$\circ \circ \quad y' = \cos^2 x - \tan x (\sin x \cos x - \cos x) = \cos^2 x$$

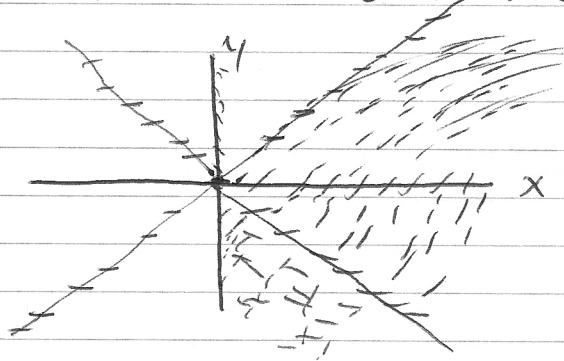
$$\left( y(0) = -1 \stackrel{?}{=} \sin(0) \cos(0) - \cos(0) = -1 \quad \checkmark \right)$$

$$\circ \circ \quad y' = \cos^2 x - \frac{\sin x}{\cos x} (\sin x \cos x - \cos x) = \cos^2 x$$

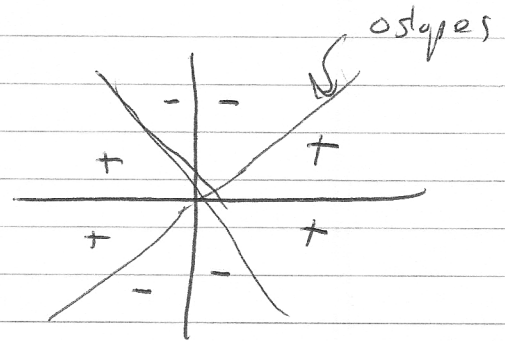
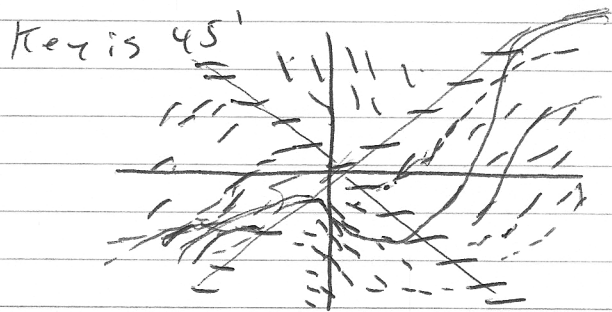
$$= \cos^2 x - \sin^2 x + \sin x \quad \checkmark$$

SAME

9)  $y' = x^2 - y^2$   
 Sketch direction of diff Equation



$y'$	$x$	$y$
0	0	0



10)  $\frac{dy}{dt} = te^y$   $y(1) = \emptyset$

~~$\int dy = \int te^y dt$~~   $y(1) = \emptyset$

$y + C =$

$\frac{1}{e^y} dy = t dt$

$\int \frac{1}{e^y} dy = \int t dt$

~~$-e^{-y} + C_1$~~   $= \frac{1}{2} t^2 + C_2$

$y(1) = \emptyset$   
 $t=1 \Rightarrow y = \emptyset$

~~$-e^{-y}$~~   $= \frac{1}{2} (1)^2 + C_2$

~~$-1$~~   $= \frac{1}{2} + C_2$

$C_2 = -1\frac{1}{2} = -1.5 = -\frac{3}{2}$

$\therefore -e^{-y} = \frac{1}{2} t^2 - \frac{3}{2}$