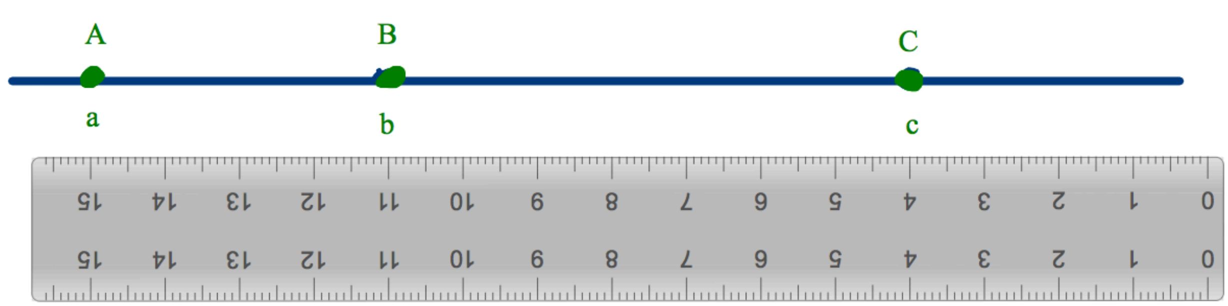
Geometry
Chapter 3 Lesson 2
Number 47

For umber 47 you are basically copying the proof on page 86 of the text, but replacing some of the values to account for the change in inequalities.

If you look at the number line diagram on page 86, your brain most likely assumes that the point "a" is smaller than the point "c", because we naturally read left to right. That particular proof where a < b < c, supports that assumption and the following proof treats the number line as if the values are less on the left side, and increasingly greater as you move to the right.
However, in number 47 of this same lesson they are asking you to approach the problem as if the larger values are on the left and things get progressively smaller as you move right. That's what the phrase "a > b > c" is meant to indicate.

From there you are just walking through the proof exactly as it is written on page 86, but switching the placement of the numbers specifically in step 3 where you have the ruler postulate. To get a positive answer to a subtraction problem, the first number in the problem has to be larger than the second. Therefore, in this step for number 47 we have to actually say "AB = a –b" that means, that if we were to add values to this number line it would look like this:



Notice that in this image I have inverted the ruler so that the larger numbers are on the left and the smaller numbers are towards the right, as indicated by the problem that the scenario should be.

Ok, now we have to realize what all of these letters are trying to accomplish. The term "AB" means "Tell me how long the segment that starts at A and ends at B is in terms of units", in order to answer that question, we can either use a ruler to measure, or we can use the values of point "a" and point "b" to determine the length. Point "a" in my diagram above is sitting on number 15, and point "b" is sitting on number 11. Common sense tells us this means that segment AB is 4 units long, but if we did not know the values of "a" and "b" (as we do not know them in problem 47) we would have to find another way to express this process. We could say "subtract a from b" as we did in the example on page 86. However, if we look at the diagram and try to apply that logic, we'll find that 11-15 = -4. That doesn't make sense, given that length cannot be measured in negative values. If something is "So many units long" the 'so many units" cannot be a negative number. So what else can we do? Well, subtraction will end up postive if we place the larger value first, and then subtract the smaller value. In the scenario where a > b > c, "b" is the smaller value. So we replace the "AB = b – a" that they had on page 86, with "AB = a – b" for number 47. It helps to think of the situation as opposite, which it is in fact, the turned around version of the same idea.

Does that help?

Then with the segment BC, we apply the same process, and say that since c is smaller than b, to get the length of the segment BC we would have to subtract the smaller value from the larger value, so BC = b – c.

From there, it is simply substitution,

We are trying to prove that AB + BC = AC using the scenario where a > b > c
A more logical way to say this is that the lengths of the line segment are the same no matter which side of the ruler you are looking at. If you walk from your house to your mailbox, and then turn around to walk from your mailbox to your house it does not matter if you were headed in the "Positive" direction (a < b < c) or the negative direction (a > b > c) the number of steps you took (which corresponds to the length of the segment) is exactly the same both times.

So in order to prove that AB + BC = AC, we have to first show what AB + BC equals in terms of the equations we came up with above. The final step, then, will be to show that AC equals the same as the AB + BC expression, and then we can make the conclusion the AB + BC = AC. That process looks like this:

AB + BC = (a - b) + (b – c)   This is simply substitution.
We simplify this equation to find that (a - b) + (b – c) = a –c
By the same process that told us AB = a –b , and that BC = b –c, we can also state that AC = a –c
So since AC = a –c AND AB + BC = a – c
We can conclude that AC = AB + BC, thus proving the theorem.